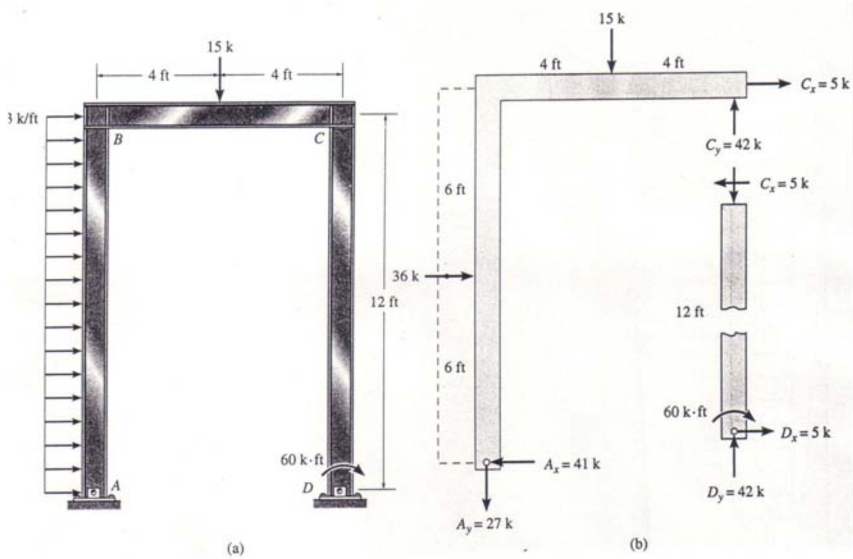
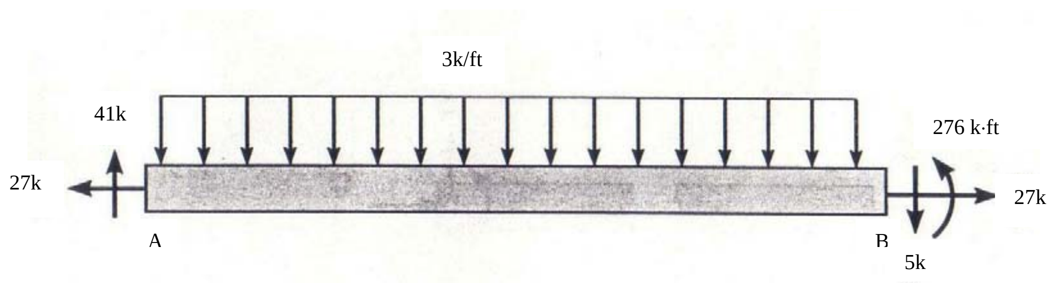
6.4 - Internal forces in frames. While the calculation of internal forces in frames follows the same principles as beams, the issue is complicated by the need to establish sign conventions for both vertical and inclined components. In addition, frames will usually have non-zero normal forces, **N**, in the components.

With respect to sign conventions, the best approach is to simply rotate the drawing in a clockwise direction until the member of interest is horizontal. Then apply the sign conventions of Section 6.1.

The frame shown below has two pinned supports at A and D and a hinge at C. Thus it is statically determinate with reactions $A_y$ = -27 k, $D_y$ = 42 k, and $D_x$ = 5 k. Note that the moment of 60 k·ft at D is an externally applied moment.




We will now determine the internal forces in member AB. To do this we will rotate the frame 90° clockwise so that AB is horizontal. Isolate AB by cutting sections at A and at B.



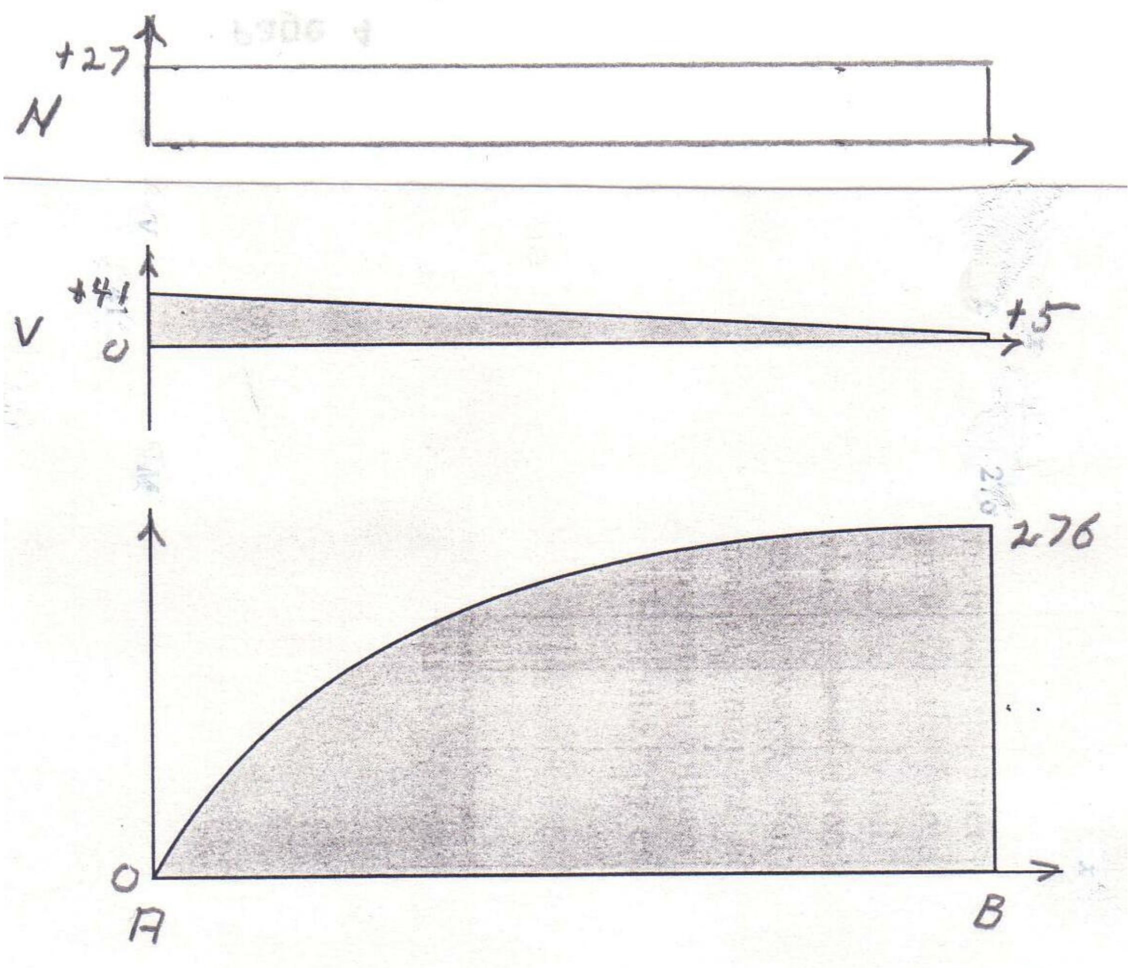

We have to be very careful about sign conventions here. First consider end A of the column. $A_y$ causes a normal force, **N**, in the member. From the point of view of our sign convention for <u>external</u> reactions, $A_y$ would be negative because it is downward; however, because $A_y$ tends to put member AB in tension, it is positive from the point of view of <u>internal</u> forces.

$A_x$ causes a shear force, **V**, in the member. From the point of view of our sign convention for <u>external</u> reactions, $A_x$ would be negative because it is to the left; however, because $A_x$ tends to rotate member AB clockwise, it is positive from the point of view of <u>internal</u> forces. $A_x$ also affects the bending moment, **M**, in the member. Because $A_x$ tends to bend the column so that it forms a bowl (after the 90° rotation), $A_x$ contributes a positive moment. Finally, the 3 k/ft uniform load contributes negative bending moment because it tends to bend the column so that it does not form a bowl.

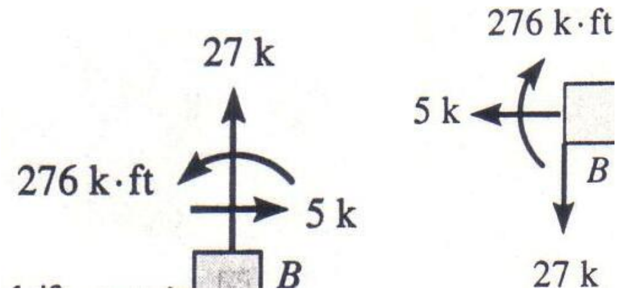
Next we will look at end B of the column. The portion of the frame that we removed when we cut the section at B exerts a normal force, **N**, of +27 k (puts tension in AB), a shear, **V**, of +5 k (tends to rotate AB clockwise), and a moment, **M**, of +276 k·ft (tends to form a bowl.

The normal force, **N**, is constant at 27 k all along the column. The equations for shear and moment are: $\mathbf{V} = 41 - 3x$ and $\mathbf{M} = 41x - 1.5x^2$

The normal force, shear force, and moment for section AB are plotted below.



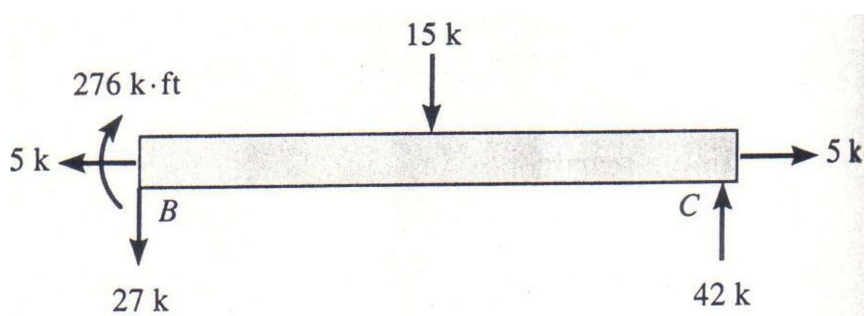
Next we will look at section BC. As the first step, we will determine how the internal forces are transferred from column AB to beam BC at point B. Specifically, the forces on the "B" end of beam BC are determined by applying Newton's third law to a section cut at joint B. The process is shown in the figure below:



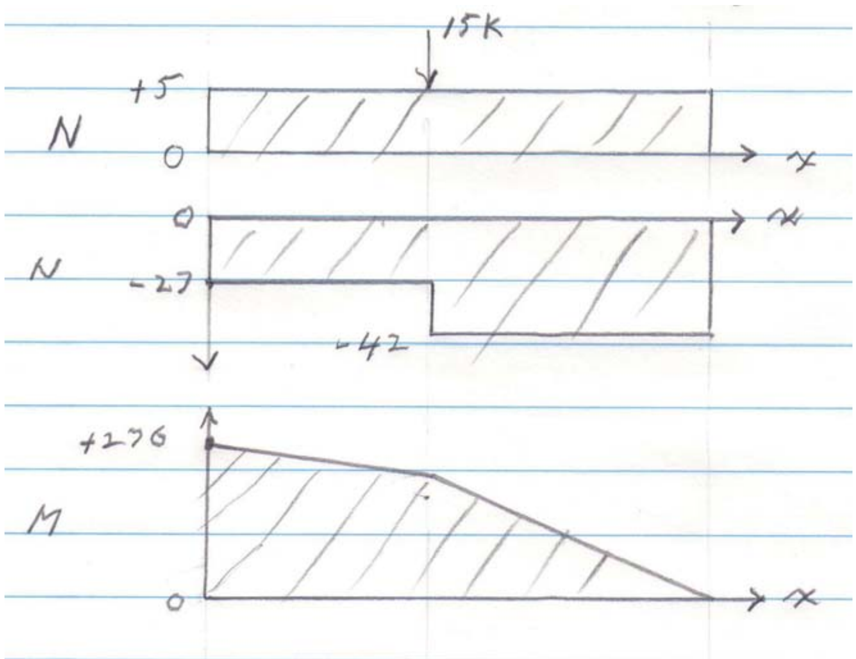

Note that the force that causes shear in column AB causes a normal force in beam BC. Moreover, the force that causes a normal force in column AB causes shear in beam BC. This, of course, is because of the 90°change in direction between these two members.

Notice particularly that a positive shear (+5 k) at B on column AB causes a positive normal force at B on beam BC (puts BC in tension). Similarly, a positive normal force (+27 k) at B on column AB causes a negative shear at B on beam BC (tends to rotate BC counterclockwise). Finally, a positive moment (+276 k·ft) at B on column AB causes a positive moment at B on beam BC (forms a bowl).

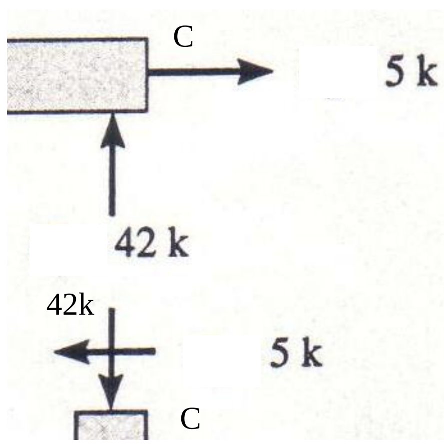
We now continue with the analysis of beam BC. We do not need to rotate BC because it is already horizontal. Isolate BC by cutting sections at B and at C. The internal forces at end B have already been determined immediately above. The reaction components at end C can be determined from the external stability of the structure. We did not do this at the very beginning of this problem, but the student should do it now as an exercise.




The normal force is again constant at +5 k all along the beam. The shear is -27 k up to the 15 k load, and is -42 k from that point to point C. The moment is given by M = +276 – 27$x$ from B to the 15 k load. It is M = +276 – 27$x$ - 15·($x$ – 4) from the load to C. The normal, shear and moment are plotted below.

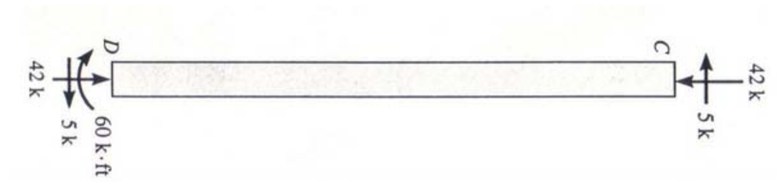



Finally, we determine the internal forces in column CD. The transfer of forces from BC to CD at joint (pin) C is shown in the figure below.



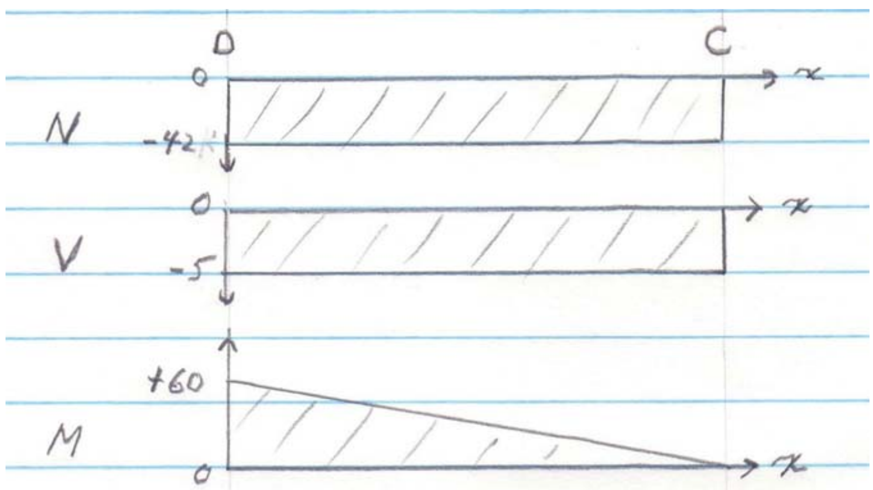

Again, we use Newton's third law to determine how the forces at end C of beam BC transfer to end C of column CD. Note that a shear of -42 k at C on BC (counterclockwise rotation) transfers as a normal force of -42 k (causes compression) on CD. Similarly, a positive normal force of +5 k (causes tension) at C on BC transfers as a shear force of -5 k (causes counterclockwise rotation) at C on CD. There is no moment to transfer because of the pinned connection.

To continue with the analysis of column CD, we need to rotate it 90° clockwise (remember – always clockwise).

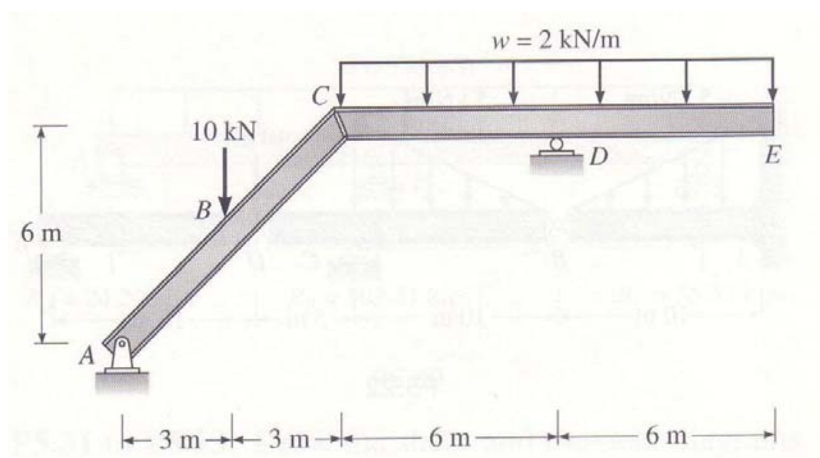



The reaction forces at D were determined at the beginning of the problem. In addition, the applied moment of 60 k·ft also acts. More specifically, at D the normal force is -42 k (puts CD in compression). The shear is -5 k (tends to rotate counterclockwise). The moment is +60 k·ft (tends to form a bowl).

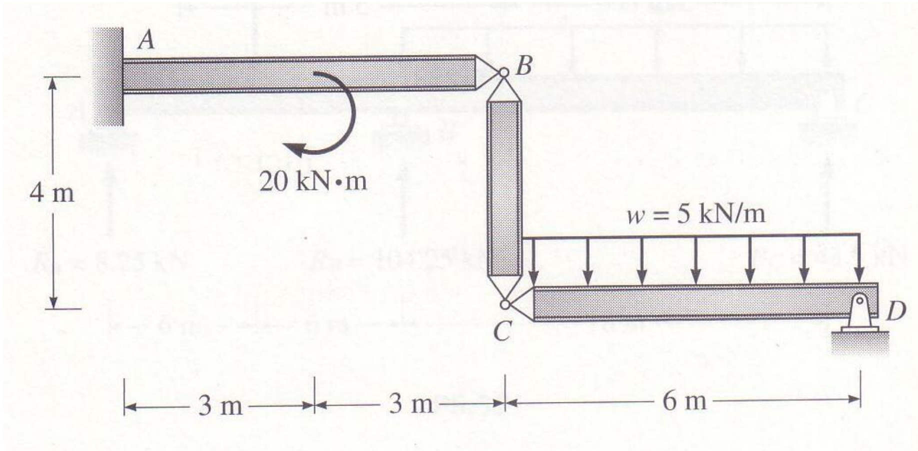
The normal force is constant at -42 k all along the section. The shear is also constant at -5 k all along the section. The moment is given by M = + 60 – 5*x* with *x* measured from D. The normal, shear and moment are plotted below.



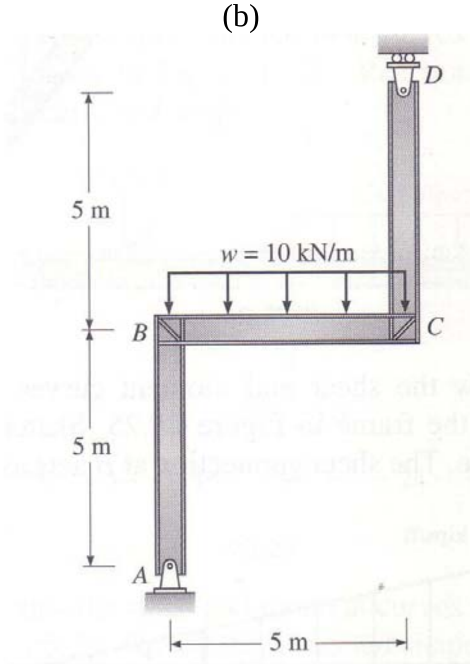

Section 6.6 - Exercises. Plot normal force, shear, and moment for the frames shown below.



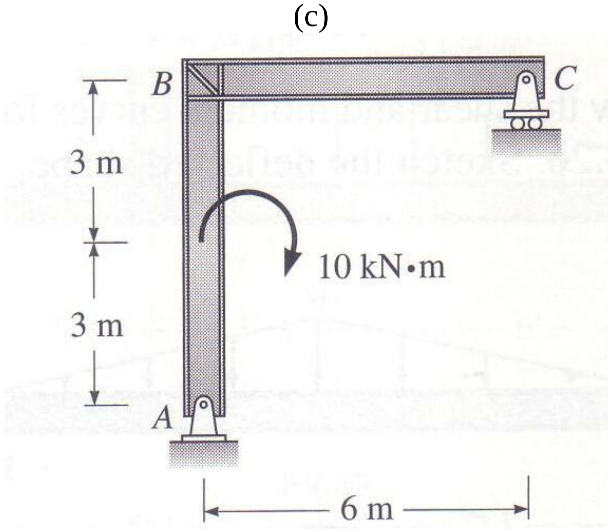

(a)


A
B
4 m
20 kN•m
w = 5 kN/m
C
D
3 m
3 m
6 m

(b)


D
5 m
w = 10 kN/m
B
C
5 m
A
5 m

(c)


B
C
3 m
10 kN•m
3 m
A
6 m

(d)