5.4 -Method of joints for computer analysis. For an internally and externally statically determinate truss there will be n unknown forces in the components plus 3 unknown reactions. There will be 2j equations from $\Sigma F_h = 0$ and $\Sigma F_v = 0$ at the nodes plus 3 equations of equilibrium for the truss as a whole. Thus

$$2j = n + 3$$

You have a choice as to whether you solve for the reactions independently or whether you just keep them as unknowns. Generally, it is best to keep them as unknown because it does not increase the number of equations to be solved significantly. Moreover, if you anticipate looking at more than one loading condition, the reaction forces will change from loading to loading.

If you decide to solve independently for the reactions, you will need to reduce the number of node equations ($\Sigma F_h = 0$ and $\Sigma F_v = 0$) by three to have the same number of equations that you have unknowns. Be careful, however, to include each unknown component force in at least one equation or you will not be able to calculate its value.

5.4.1 – It is very important to define and consistently follow a rigorous protocol for sign conventions. Here is a consistent set of conventions:

(1) For vertical forces, take upward as positive,
(2) For horizontal forces, take to the right as positive,
(3) Assume all reactions to be in the positive (upward or to the right) direction, and
(3) Assume all components to be in tension.

If you follow this sign convention, all forces that come out positive will represent components in tension. All forces that come out negative will represent components in compression. Moreover, all positive vertical reactions will be upward, and all positive horizontal reactions will be to the right. All negative reactions will be downward (vertical) or to the left (horizontal).

In addition, it is best to measure all angles off of the horizontal. If you do this, all horizontal components of sloping bars will involve the cosine of the angle, and all vertical components will involve the sine. Thus, you do not have to always be stopping to figure out which trig function to use.

Fig. 5.4.1 shows a simple truss for analysis. Two approaches are shown. In the first approach, the reactions are solved for independently, and the component forces are then determined. In the second approach, the reaction forces are lumped in with the component forces as unknowns.

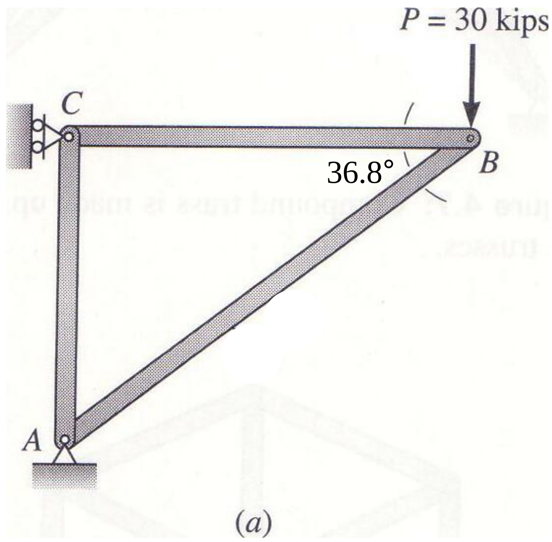


Fig. 5.4.1 - Sample truss for computer analysis.

Approach 1: Solving for reactions independently.

$\Sigma F_v = 0$ gives $A_y = 30$kips

$\Sigma M_A = 0$ gives $C_x = -40$kips

$\Sigma F_h = 0$ gives $A_x = 40$kips

Writing an equation for $\Sigma F_h = 0$ at C gives:

$-40 + F_{CB} = 0$

Writing an equation for $\Sigma F_h = 0$ at B gives:

$-F_{CB} - \cos(36.8°)F_{AB} = 0$

Writing an equation for $\Sigma F_v = 0$ at C gives:

$-F_{AC} = 0$

We now have three equations and three unknowns:

$$\begin{cases} F_{CB} - 40 = 0 \\ -F_{CB} - \cos(36.8°)F_{AB} = 0 \\ -F_{AC} = 0 \end{cases}$$

These can be solved simultaneously for F_{CB} = 40 , F_{AC} = 0, and F_{AB} = -50. Note that, because the number of equations/unknowns is only three, the system of simultaneous equations is trivial to solve. This will not always be the case.

Approach 2: Keeping the reactions as unknowns.

Writing an equation for $\Sigma F_h = 0$ at C gives: $C_x + F_{CB} = 0$

Writing an equation for $\Sigma F_v = 0$ at C gives: $-F_{AC} = 0$

Writing an equation for $\Sigma F_h = 0$ at B gives: $-F_{CB} - \cos(36.8°)F_{AB} = 0$

Writing an equation for $\Sigma F_v = 0$ at B gives: $-30 - \sin(36.8°)F_{AB} = 0$

Writing an equation for $\Sigma F_h = 0$ at A gives: $A_x + \cos(36.8°)F_{AB} = 0$

Writing an equation for $\Sigma F_v = 0$ at A gives: $A_y + F_{AC} + \sin(36.8°)F_{AB} = 0$

Writing these equations collectively and rearranging terms into standard order gives the system:

$$\begin{cases} F_{CB} + C_x = 0 \\ -F_{AC} = 0 \\ -F_{CB} - \cos(36.8°)F_{AB} = 0 \\ -\sin(36.8°)F_{AB} = 30 \\ \cos(36.8°)F_{AB} + A_x = 0 \\ \sin(36.8°)F_{AB} + F_{AC} + A_y = 0 \end{cases}$$

These can be solved for F_{AB} = -50, F_{BC} = 40, F_{AC} = 0 A_x = 40, A_y = 30, and C_x = -40.

5.4.2 – Solving the simultaneous equations. Solution of systems of simultaneous equations is treated in detail in CE406. Only a brief overview will be presented here. The above equations can be written in matrix form as:

[A]{x} = {b}

where

$$[A] \equiv \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -\cos(36.8°) & 0 & 0 & 0 \\ 0 & 0 & -\sin(36.8°) & 0 & 0 & 0 \\ 0 & 0 & \cos(36.8°) & 0 & 1 & 0 \\ 0 & 1 & \sin(36.8°) & 0 & 0 & 1 \end{bmatrix} \quad \{x\} \equiv \begin{bmatrix} F_{CB} \\ F_{AC} \\ F_{AB} \\ C_x \\ A_x \\ A_y \end{bmatrix} \quad \{b\} \equiv \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 30 \\ 0 \\ 0 \end{Bmatrix}$$

To solve this system we need to invert the [A] matrix to obtain the form:

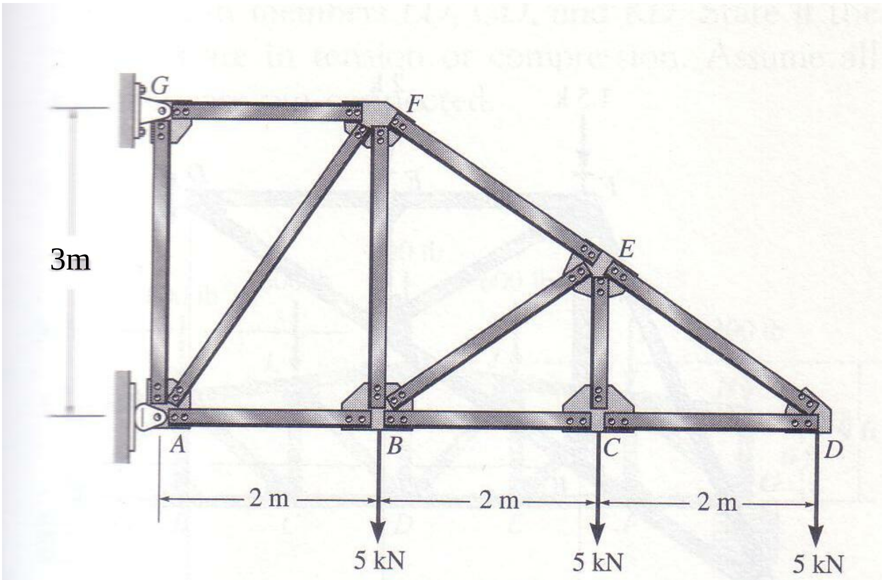
$$\{x\} = [A^{-1}]\{b\}$$

Finding $[A^{-1}]$ is not a simple process, but software like MATLAB, Excel, etc. have routines to do this. The techniques for solving these equations using Excel can be found in ExcelMatrix.pdf on the course home page. One word of caution – Excel wants angles in radians. The RADIANS() function does the job. Thus get the cosine of 30° you would type:

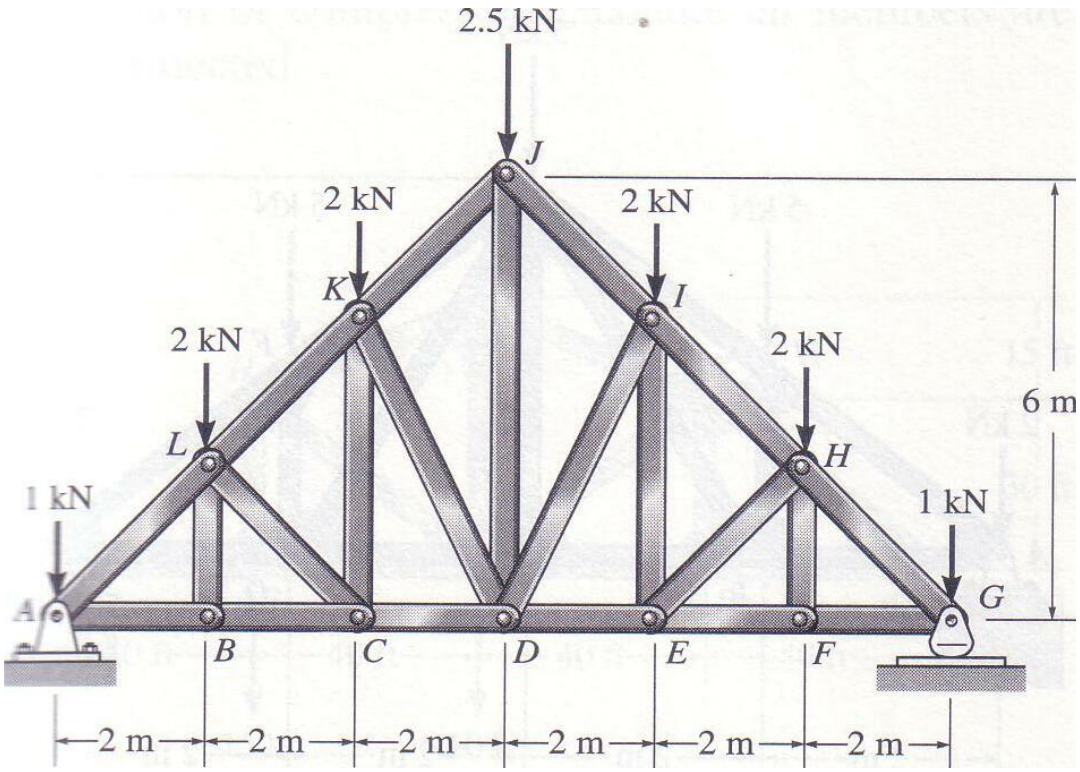
=cos(radians(30))

Having found $[A^{-1}]$, you can then use other routines in MATLAB or Excel to do the matrix multiplication $[A^{-1}]\{b\}$. This will give you the component forces and reactions, {x}.

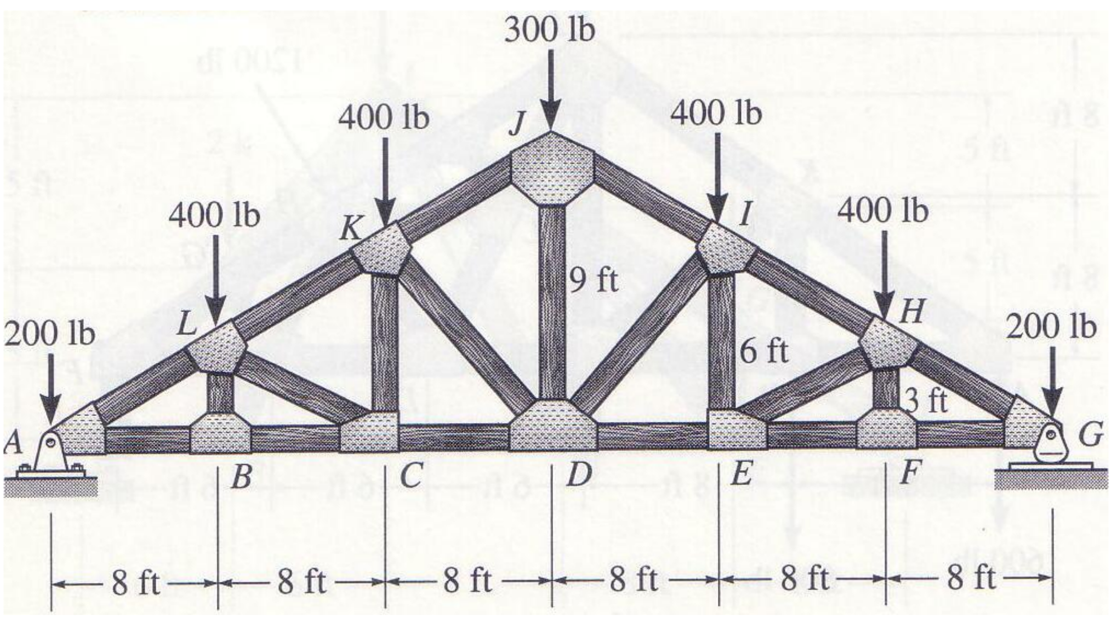
5.4.3 - Exercises. Use matrix methods in Excel to solve for the reactions and forces for the trusses shown below.



(a)


2.5 kN
2 kN
2 kN
2 kN
2 kN
1 kN
1 kN
J
K
I
L
H
A
G
B
C
D
E
F
6 m
2 m
2 m
2 m
2 m
2 m
2 m

(b)


300 lb
400 lb
400 lb
400 lb
400 lb
200 lb
200 lb
J
K
I
L
H
A
G
B
C
D
E
F
9 ft
6 ft
3 ft
8 ft
8 ft
8 ft
8 ft
8 ft
8 ft

(c)