8.1 - Influence lines. Previous sections of these notes have dealt with how component forces (normal, shear, moment) vary with position along a loaded structure. In this section, we will look at how components forces at a single location vary as a load moves across the structure. A graph of how a restraint reaction or component force varies with position of applied load is called an influence line. Influence lines are always linear plots, although there are often changes in the slopes and discontinuities in the values.

The process for constructing influence lines is relatively straight forward:

(1) pick a reaction or component for which you want to construct the influence line, and
(2) evaluate the force as you successively move a load of unit magnitude across the structure.

Nevertheless, by understanding a little about the character of influence lines, you can greatly simplify the process.

First, consider the influence line for the left reaction, R_A, of the beam shown in Fig. 8.1.1:

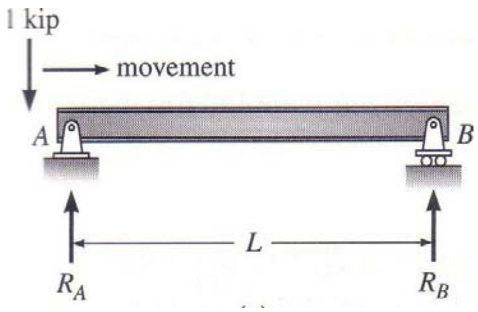
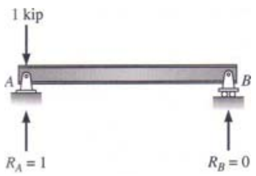
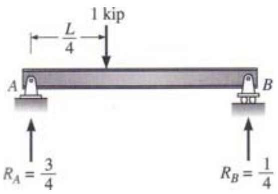
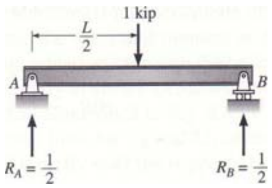


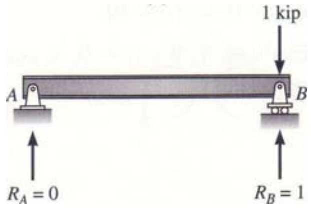
Fig. 8.1.1 – Beam with 1 kip load moving across it.

When the load is directly over R_A, its full force is carried by R_A. Thus, the value of the influence line is 1.0. When the load is located at L/4, R_A carries 3/4 of the load, and when the load is located at L/2, R_A carries 1/2 of the load. Finally, when the load is located directly over R_B, reaction R_A carries none of the load.

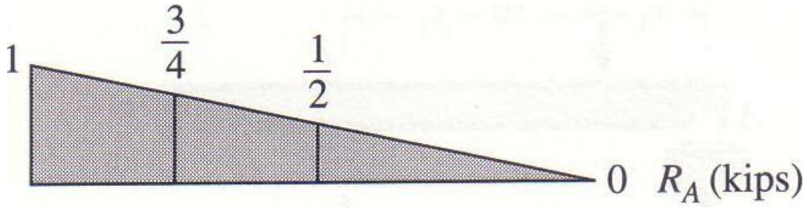








The influence line is plotted below.



This influence line can be plotted with a lot less effort if we understand from first principles that the value of R_A varies linearly from 1.0 to 0 as the load moves from directly over R_A to directly over R_B. Thus we did not really have to make any "calculations".

As a second example, consider the influence lines for shear and for moment at B on the beam shown in Fig. 8.1.2.

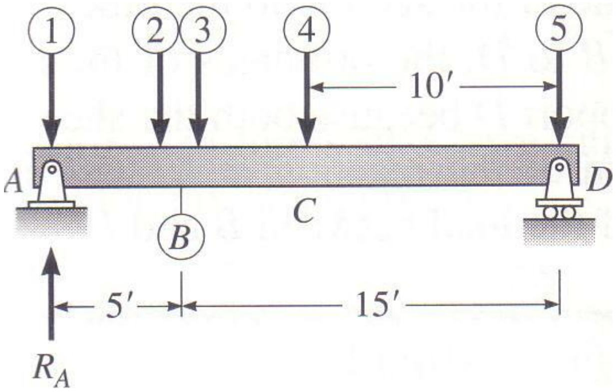
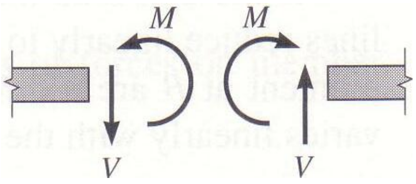


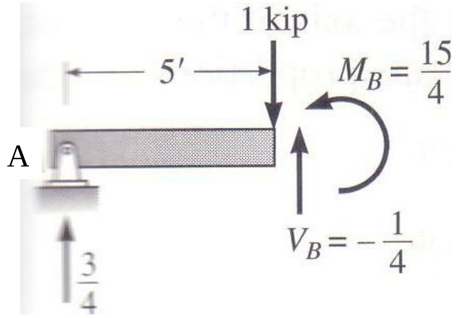
Fig. 8.1.2 – Beam for analysis.

We start by reasoning out the characteristics of the influence line. In particular, the forces (shear and moment) are likely to vary uniformly between A and B. We might expect changes as the load moves from just to the left of B (point 2) to just to the right of B (point 3). Finally, we might expect things to vary uniformly between B and D. Moreover, we would expect the forces to be zero when the load is located directly over A and directly over B because the load would then be carried directly into the reactions.

Thus, the only two analyses that we really need to do is to calculate the shear and the moment just to the left of and just to the right of B. To begin, we calculate R_A when the unit load is just at B to be 3/4 directed upward. The shear and moment acting at B are shown below.

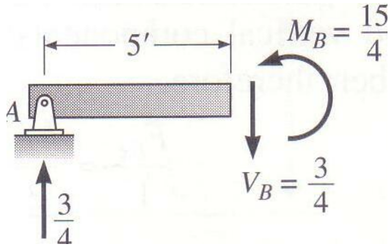


With the load just to the left of B and looking at B, the free-body diagram is



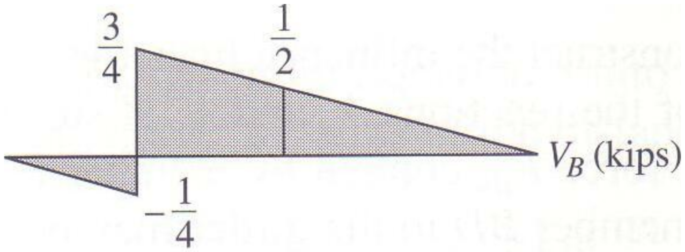
The shear is V = +3/4 – 1 = -1/4. The moment at B is M = 3/4(5) = +15/4. Thus the shear varies linearly from 0 when the load is at A to -1/4 when the load is just to the left of B.

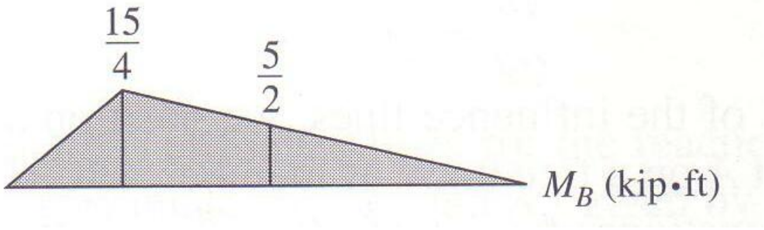
As the load moves just to the right of B, the following free-body holds.



The shear is V = +3/4. The moment is M = 3/4(5) = +15/4, as before. Thus as the load passes across B, there is a discontinuity in the shear, its value going from -1/4 to +3/4. The moment does not have a discontinuity at B, however, as we will see next, it does change slope.

When the load is at D, as previously reasoned, both the shear and moment at B are zero. The influence lines for shear and moment at B can now be drawn:





8.2 - The Muller-Breslau Principle. In the late 1800’s Muller-Breslau discovered a principle that allows us to determine the shape of an influence line by noting the deformed shape of the structure when it is appropriately altered. In particular, if a reaction or force component is at a particular location, the structure is altered so that the reaction or force component is reduced to zero at that point. For example, if you wish to find the influence line for moment at a particular location, artificially insert a pin at that point and observe the deformed shape of the structure. More, completely, the principle states that:

> The ordinates of an influence line for any force are proportional to the deflected shape of the structure produced by removing the capacity of the structure to carry the force and then applying a positive force corresponding to the restraint removed that induces a deflection in the structure.

As an example, consider the beam shown in Fig. 8.2.1.

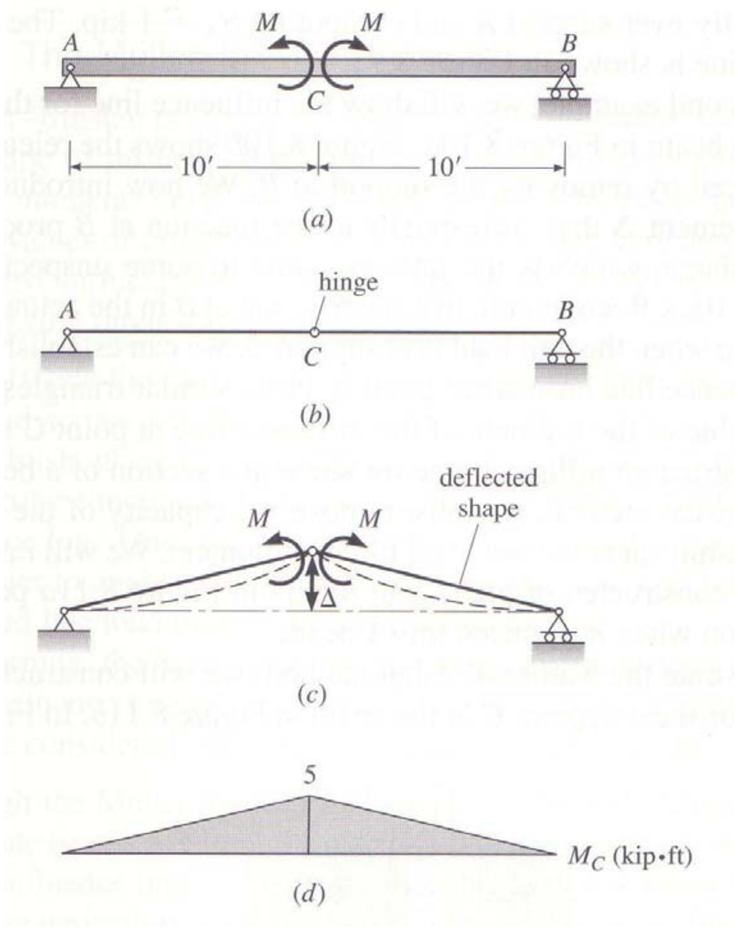


Fig. 8.2.1 – Muller-Breslau for moments at C.

If we wish to determine the influence line for moment at C, we artificially introduce a hinge at C and apply a positive internal moment at C. The deflected shape of the structure qualitatively gives the influence line. We need only calculate one value of moment (the moment at B when the load is at B) to quantify the influence line.

More examples follow:

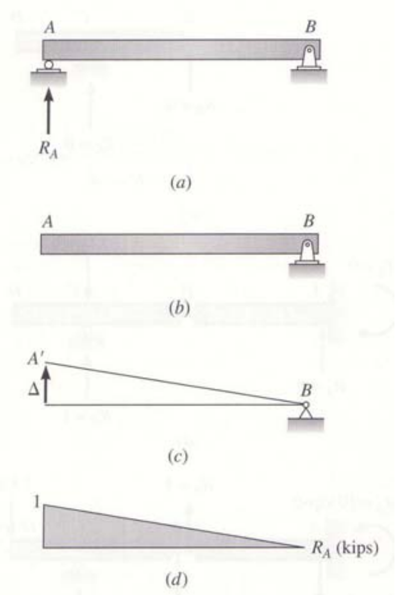


Fig. 8.2.2 - The influence line for the vertical component of R_A is obtained by releasing the vertical restraint and applying a positive, vertical external force at A.

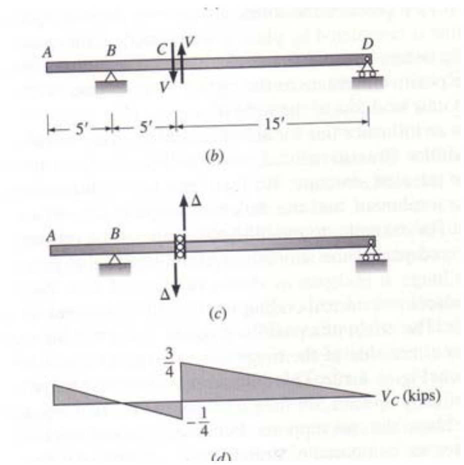


Fig. 8.2.3 - The influence line for shear at C is obtained by releasing the ability to withstand shear at C (introducing a vertical roller) and applying a positive internal shear at C.

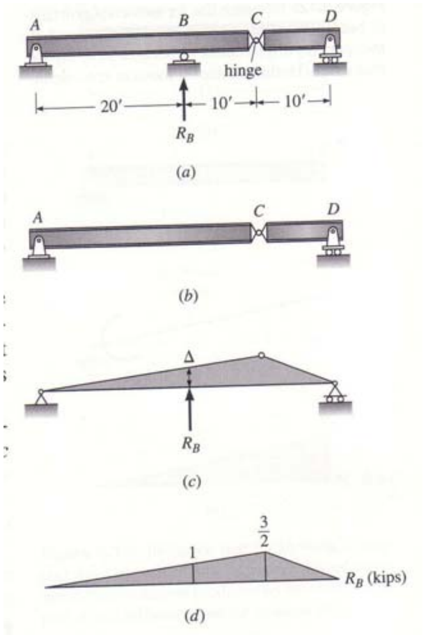


Fig. 8.2.4 - The influence line for R_B is obtained by releasing the vertical restrain at B and introducing a positive, vertical external force at B.

8.3 - Influence lines for uniform loads. The influence lines that we have constructed for unit concentrated loads on structures can also be used to determine the forces cause by uniform loads. The effect of uniform loads can be viewed as the cumulative effect of a series of unit loads successively apply over the extent of the uniform load whose effect we desire to calculate.

More formally, consider the uniformly loaded beam segment and the associated influence line shown in Fig. 8.3.1.

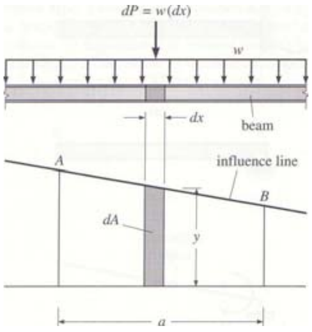


Fig. 8.3.1 – Uniform loads.

In general, to evaluate a function, *F*, resulting from a uniform load, w, acting over length, *a*, of a beam, we can introduce a series of infinitesimal forces, *dP*. Then we can sum over incremental functions, *dF*, produced by these infinitesimal forces. The force *dP* produced by a uniform load, *w*, acting over beam segment *dx* is given by

$$dP = w\,dx \qquad (1)$$

The increment in the function *dF* produced by *dP* is

$$dF = (dP)\, y \qquad (2)$$

where *y* is the ordinate of the influence line. Putting eq. (1) into eq. (2) gives

$$dF = w\, dx\, y \qquad (3)$$

To determine the effect of the uniform load placed between A and B we integrate:

$$F = \int_A^B dF = \int_A^B w\, dx\; y = w\int_A^B y\, dx \qquad (4)$$

Eq. (4) represents the area under the influence line between A and B multiplied by *w*.

As an example, consider the uniformly loaded beam shown in Fig. 8.3.2.

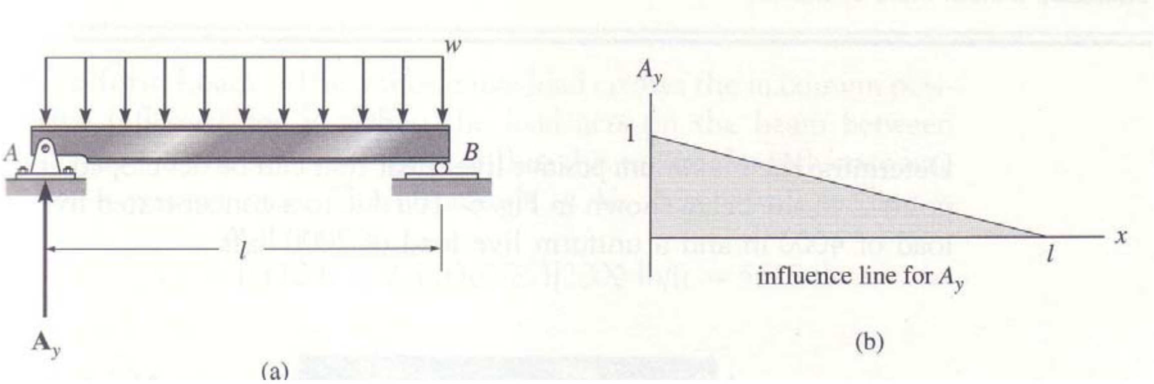


Fig. 8.3.2 - Uniformly loaded beam and influence line for A_y.

The area under the influence line for the region over which the uniform load is applied is

$$\text{Area} = \tfrac{1}{2} l \cdot 1$$

and $$A_y = \tfrac{1}{2} w\, l$$

8.4.1 - Using influence lines for superposition. Because the structures that we analyzing are linear, the effects of multiple loadings can be determined by superposition. Superposition is accomplished by adding the effects of individual loadings.

As an example, consider the beam and its associated influence line for moment at C shown in Fig. 8.4.1.

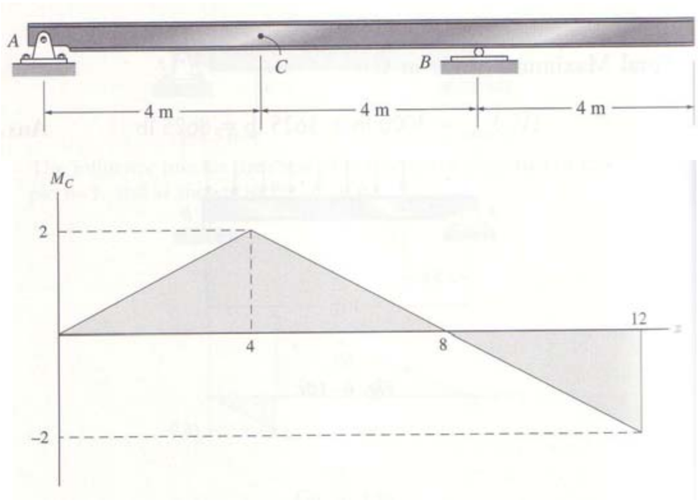
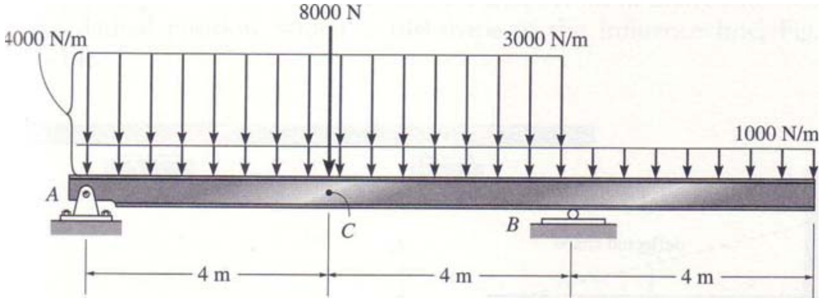


Fig. 8.4.1 - Beam and influence line.

The beam will be loaded by a concentrated 8000 N load at C, by a uniform load of 3000 N/m between A and B, and the weight of the beam itself viewed as a uniform load of 1000 N/m over the whole beam.



The moment at C can be calculated by adding the three individual contributions to moment:

$$M_C = 8000(2) + 3000(.5)(8)(2) + 1000(.5)(8)(2) + 1000(.5)(4)(-2) = 44 \text{ kN-m}$$

[8 k conc. + 3 k uniform + 1 k uniform 0 to 8 ft – 1 k uniform 8 to 12 ft]

8.5.1 – Using influence lines to calculate maximum forces. When designing structures subjected to live loads, we must determine what configurations of live load will cause the highest internal forces at critical locations in the structure. For example, if a long beam contains a connection at some point along its length, you might be interested in calculating the maximum moment and shear at the connector.

Influence lines provide a good means to accomplish this. Consider, for example, the beam shown in Fig. 8.4.1 of the previous section. Assume that we are interested in determining the loading configuration that would give the maximum positive moment at

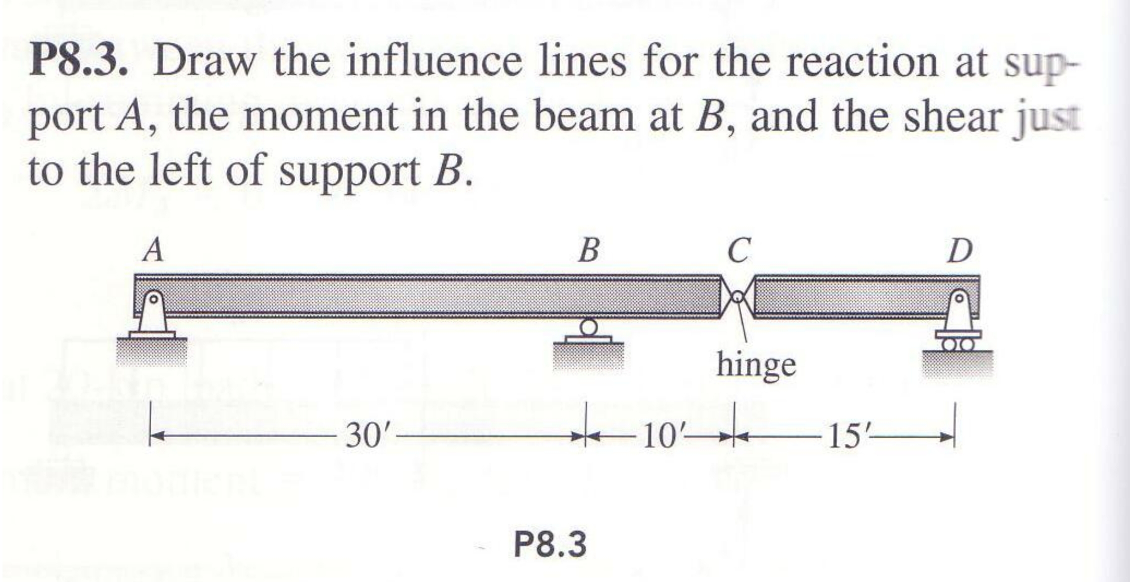
C and the configuration that would give the maximum negative moment. By examining the influence line, we see that loadings from 0 to 8 feet contribute to positive moment at C. Conversely, loadings from 8 to 12 feet contribute to negative moments. Assume that we have control over the placement of the 8 kN concentrated load and over the extent and placement of the 3 kN/m uniform load. (We will assume that we do not have control over the 1 kN/m uniform load because it represents the weight of the beam.)

The placement of the 8 kN load directly at C is the optimum placement for it if we want to check for positive moment because the influence line is most positive at C. Moreover, placing the 3 kN/m uniform load over the full A to B extent of the beam induces the highest bending moment at C because the influence lines is always positive between A and B. Thus the loading that was used for the example problem in Section 8.4.1 actually represents the configuration that would give the highest positive moment at C. The value of the highest positive moment is 44 kN-m.

To obtain the highest negative moment at C, we would have to move the 8 kN concentrated load to the end of the cantilever portion at the 12 m mark. We would also move the 3 kN/m uniform load to extend over the 4 m segment from B to the end of the cantilever portion. In this case, the maximum negative moment at C would be:

$$M_C = 3000(.5)(4)(-2) + 8000(-2) + 1000(.5)(8)(2) + 1000(.5)(4)(-2) = -24 \text{ kN-m}$$

8.6 - Exercises.

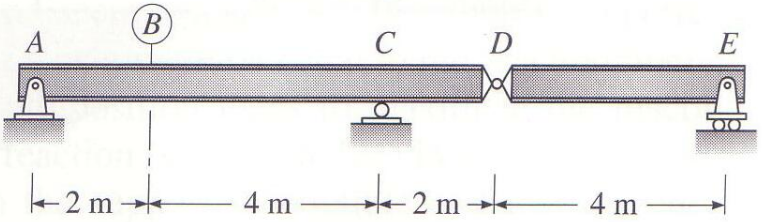


P8.3. Draw the influence lines for the reaction at support *A*, the moment in the beam at *B*, and the shear just to the left of support *B*.

P8.3

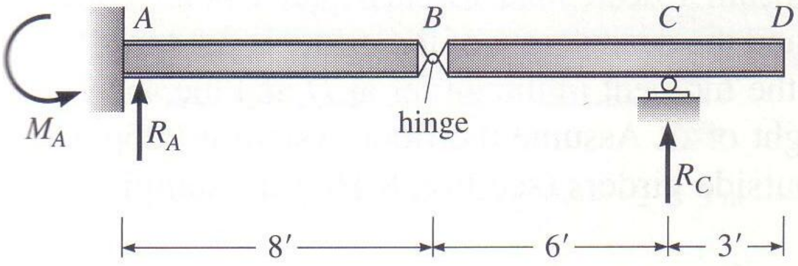
(a)

P8.4. For the beam in Figure P8.4, draw the influence lines for the reactions at *A*, *C*, and *E* and the shear and the moment at *B*. Determine the maximum value of each reaction (both positive and negative) if the beam is subject to a concentrated load of 20 kN and a 1.8 kN/m uniform load of variable length.



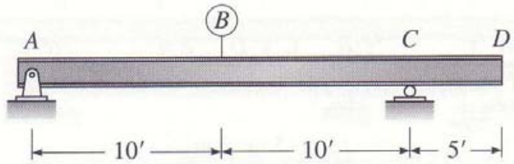
(b)

P8.5. (*a*) Draw the influence lines for reactions M_A, R_A, and R_C of the beam in Figure P8.5. (*b*) Assuming that the span can be loaded with a 1.2 kips/ft uniform load of variable length, determine the maximum positive and negative values of the reactions.



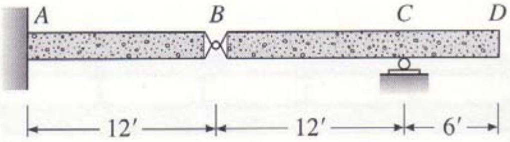
(c)

P8.8 to **P8.11.** Using the Müeller–Breslau principle, sketch the shape of the influence lines for the reactions and internal forces noted below each structure.



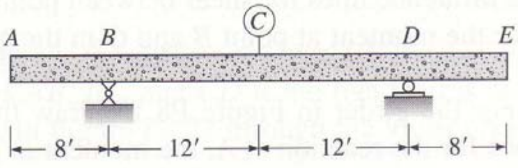
V_A, M_B, M_C, and R_C

P8.8



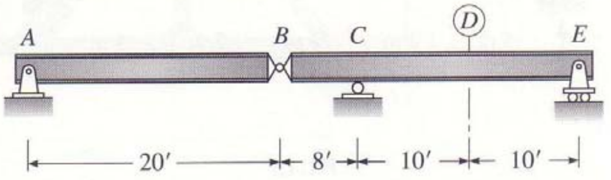
M_A, R_C, and V_C (to right of support C)

P8.9



R_B, V_C, M_C, and V_B (to right of support)

P8.10



R_A, R_C, M_D, and V_D

P8.11

(e)

8.7 – Influence line for trusses. Influence lines can be drawn for forces in truss components; however, we must first look at how loads are transmitted to the truss. A truss can only have loads applied at the pins, therefore the structure must have some form of decking so that the load can move across the truss. This decking is usually applied along the bottom cord of a bridge truss as shown in Fig. 8.7.1.

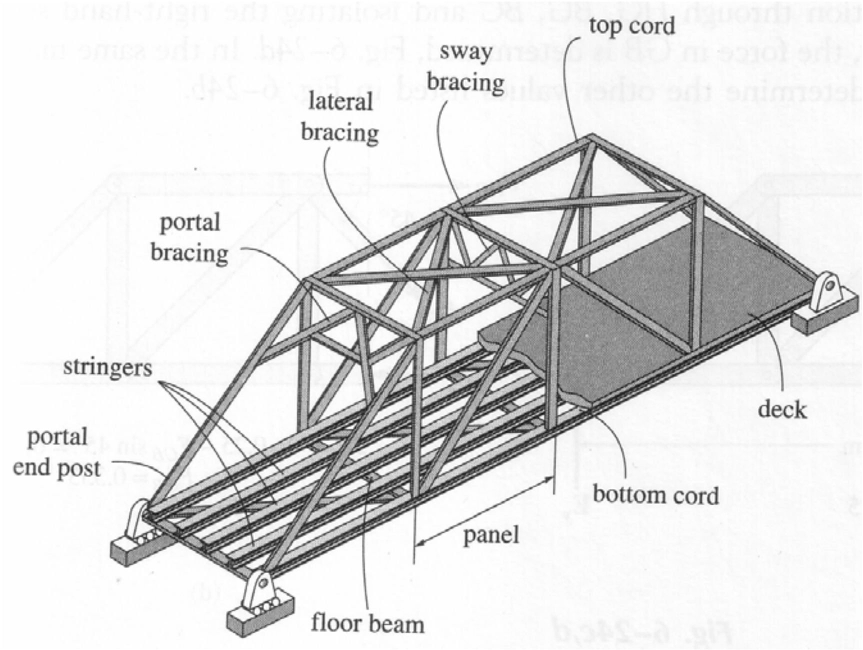
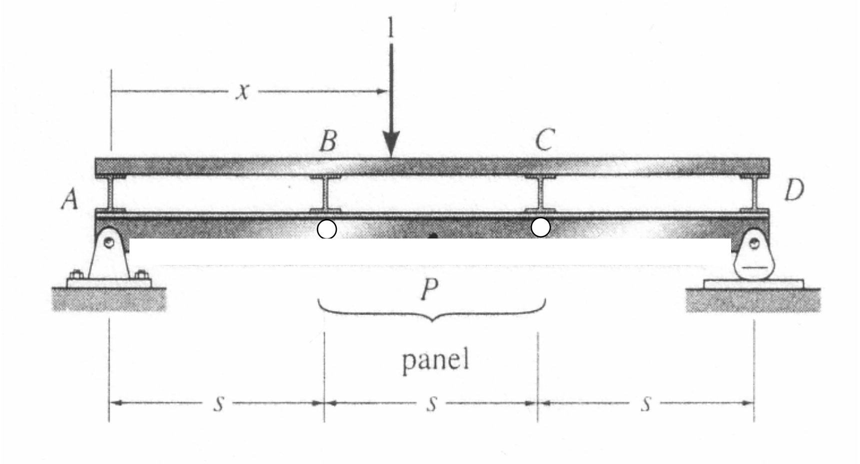


Fig. 8.7.1 – Bridge deck transmitting load to bottom cord of truss.

Fig. 8.7.2 shows a side view of a typical bridge truss with main support pins at A and D, with bottom cord ABCD, and with floor beams tied to the bottom cord pins at A, B, C, and D.



The deck is assumed to be flexible so that a load between, say, B and C is proportioned entirely between floor beams B and C. (If you wish, you can imagine the deck panels to be pinned at B and C.) Fig. 8.7.3 shows the relevant components and dimensions.

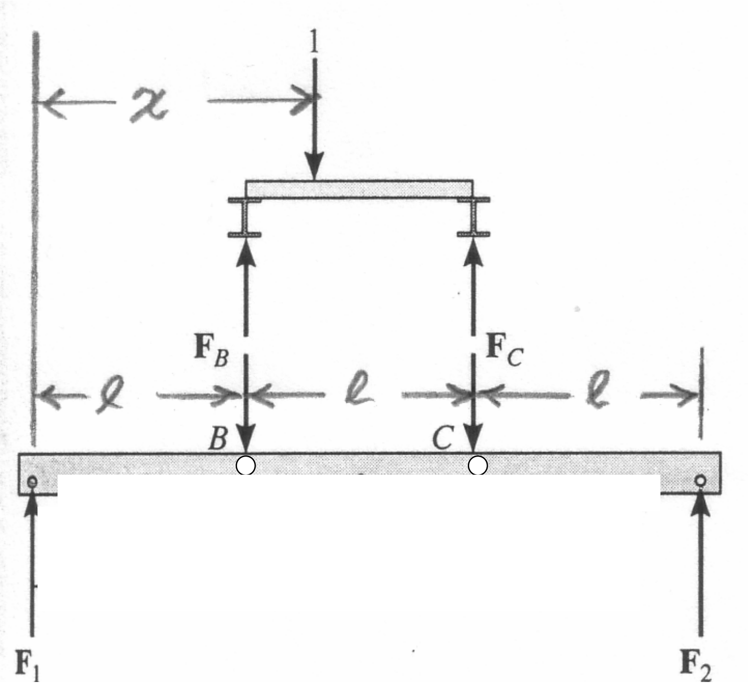


Fig. 8.7.3 – Details of load transmission between deck and bottom cord members.

The force F_B is given by:

$$F_B = \frac{2\ell - x}{\ell} \qquad (1)$$

The force F_C is given by:

$$F_C = \frac{x - \ell}{\ell} \qquad (2)$$

The reaction F_1 is given by:

$$F_1 = \frac{3\ell - x}{3\ell} \qquad (3)$$

The reaction F_2 is given by:

$$F_2 = \frac{x}{3\ell} \qquad (4)$$

Eqs. (1), (2), (3), and (4) all vary linearly in *x*, indicating that the forces in any and all truss components linearly with *x*.

If this is the case, the influence line for the force in any truss member can be determined by: (1) calculating the force with the unit load at B, (2) calculating the force with the unit load at C, and (3) interpolating linearly between B and C to construct the influence line.

This can be generalized to conclude that the influence line for the force in any member in the truss can be calculated by determining the force with the unit load at A, B, C, and D, respectively, and interpolating linearly between these values.

As an example, consider the truss shown in Fig. 8.7.4 below.

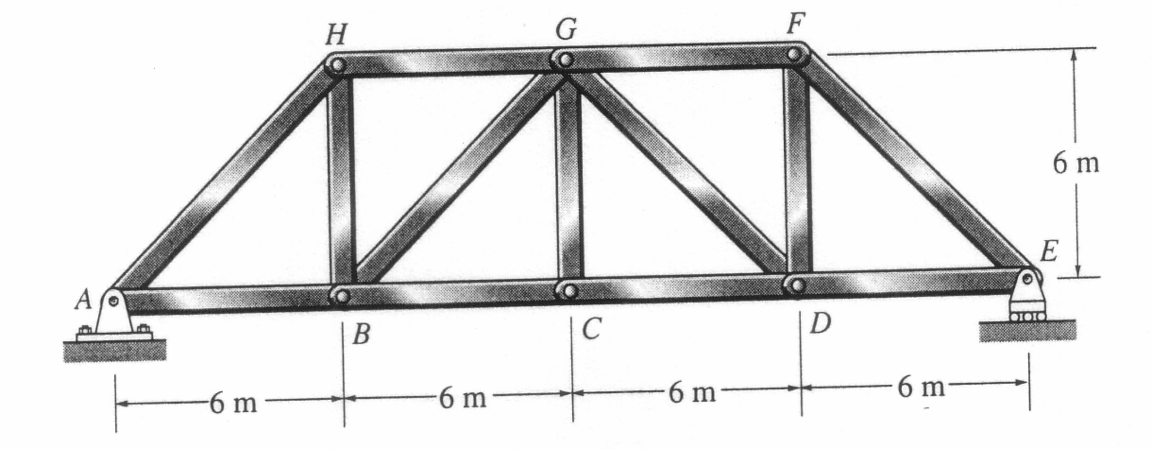


Fig. 8.7.4 – Truss for which influence line for force in GB is to be determined.

Using any method desired (for example by cutting a section through HG, BG, and BC), the student can show that the force in GB has the influence line shown in Fig. 8.7.5 below.

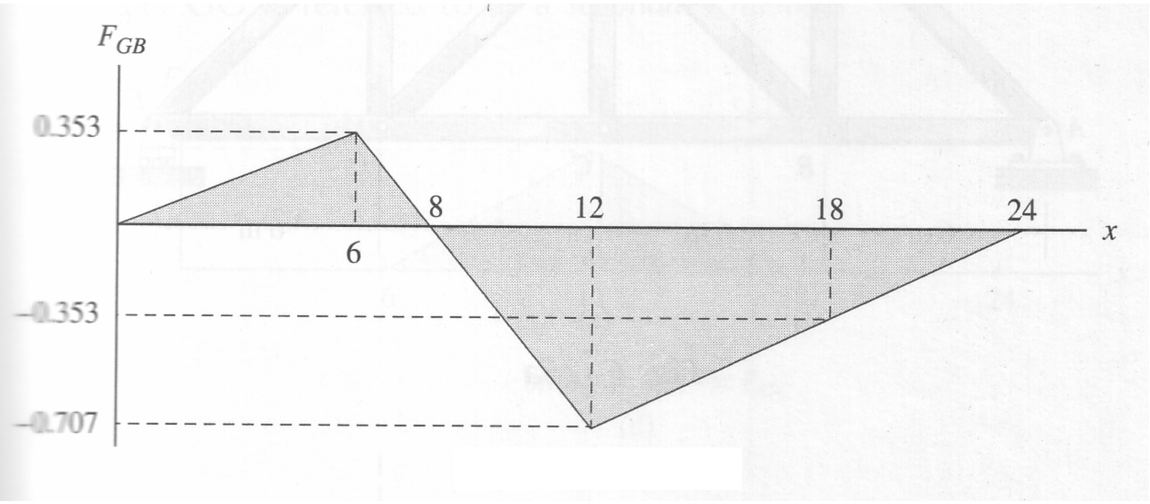


Fig. 8.7.5 – Influence line for force in GB.

Influence lines are easily done if a matrix method solution is available for the truss. The effect of locating the unit force at successive pins along the cord can be calculated by altering the {b} matrix appropriately. The influence line ordinates for any (or, for that matter, all) truss members can then be calculated. The matrix equations in the spreadsheet will adjust automatically as successive changes are made in the {b} matrix.